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Trigonometric Solutions Using Sine Quadrant

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Abstract

In astronomical calculations, the sine quadrant is an important tool apart from the use of astrolabe (asturlab). In the Malay World, the use of the sine quadrant or also referred to as Rubu[c] Mujayyab is more significant compared to astrolabe. The sine quadrant is a quadrant-shaped plate, which contains grid lines of the scale of trigonometric functions, by which readings and solving trigonometric equations can be done. This equipment can also be used to measure the altitude of celestial and other landscape objects. Hence, the sine quadrant functions as a mathematical tool that can solve trigonometric calculations. The research presented in this paper relates to the influence of the use of mathematical tools of Islamic civilization in the Malay World, with special emphasis on mathematical calculations using the sine quadrant. With references to a number of manuscripts and printed material relating to pre modern astronomy, this study found that the steps in solving trigonometric calculation in astronomy are explained descriptively using the sine quadrant. The operations in this descriptive form are then matched with basic trigonometry in mathematics. Examples of solving trigonometric operations shown in this discussion, prove the effectiveness of the use of the sine quadrant.

Keywords: Descriptive mathematics; Sine quadrant; Trigonometry

1. Introduction

Calculation and observation activities in astronomy are closely intertwined. Astronomers measure the position of the Sun to calculate time or vice-versa. Some among them measure the positions of the stars to determine the points of the Earth. Some also measure the planetary position to predict the future occurrence of celestial phenomena, or to verify the mathematical model of the celestial movement formula which they created. Observations to gather empirical data require special instruments, in the same way as calculation requires the aid of calculation tools. When these two requirements are combined, it becomes an astronomical instrument rich with concepts of geometry, arithmetic and trigonometry. Among the instruments that meets this purpose are the astrolabe and sine quadrant. The purpose of this study is to adapt the use of sine quadrant as a solution to solve trigonometric equations, which are usually performed by using logarithm tables or scientific calculators. This can only be done when mathematical descriptive elements in the treatise have been transformed into quantitative formulae with specific symbols and notations.

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2. Sine Quadrant as a Mathematical Tool

In the early period, the astrolabe was a globe which displayed the formation and position of a constellation of stars that became myths of society at the time. According to Torode (1992), the earliest astrolabe invented by Ptolemy (130 AD) contained several circles which represent the spherical projection of the Sun and the Earth and several stars. From civilization to civilization, more elements were added to the globe such as tracks of the Sun and Moon's position and the difference in size between the stars. As the globe type of astrolabe was limited of components, it was spread out to become a plane astrolabe. This allowed the addition of the latitude plate, the horizontal plates which represent the local horizon of the Earth, the equator plane and the tools to measure the angles of two points. According to King (1987) and Nasr (1976), other than the circle projected celestial sphere, the astrolabe can also be used for the measurement and trigonometric calculation of numerical solutions. From the plane astrolabe, a part of its section is taken as the sine quadrant. Compared with the astrolabe, the sine quadrant is more limited in its components and function. In the Malay World during the pre-modern period, the sine quadrant was widely used as a mathematical and astronomical tool for astronomers. In this study, pre-modernization refers to the period before the widespread influence of European mathematics such as the use of trigonometric tables or logarithmic tables.

A study of the collection of astronomical manuscripts of pre modern Malay World reveal that the sine quadrant treatise was among one of the most common subjects. Among the earliest manuscripts discovered and used in the Malay World in relation to the sine quadrant is the manuscript *al-Jauharun Naqiyah fil A^{c}amali Jaibiyah*, compiled by Syeikh Ahmad bin Abdul Latiff al-Minangkabaui al-Khatib, in the Arab and Malay language version around the 1890s. In referring to Baharrudin (2007), this manuscript contains a part of Sibt al-Maridini treatise on 15th. Century on the use of sine quadrant. Our study reveals that up until the 1940s there were at least ten (10) copies related to the sine quadrant in circulation. The main content usually found in copies related to the sine quadrant are as follows;

- Identification of the parameters and components in the sine quadrant
- Method for measuring the height (altitude) of the Sun
- To arrange the zodiac calendar
- To determine the Sun's coordinates
- Methods of measuring and reading time
- Calculating prayer times
- Determination of the latitude, longitude and local meridian
- Calculating the direction of the qiblah to Mecca
- Measuring the height and depth of objects and measuring the width and the sub-division of land

This shows that the sine quadrant is not only used to produce data and astronomical calculations, but it was also used in agricultural and engineering works. In general, the sine quadrant is the only pre modern mathematical instrument that can be used as a measurement instrument, to assist in calculations and as a tool which provides astronomical information. As a measurement tool, the sine quadrant is used to measure angles which are then changed in other parameters. The transformation of one form of value to another form can be solved using sine quadrant.

2.1 Design of the Sine Quadrant

In the development of Islamic civilization and science in the Malay World, the sine quadrant tool is better known as *rubu^c mujayyab*. The word '*rubu^c*' means quadrant, while the word '*mujayyab*' refers to grid point of sine ratio. Thus [3], states that the sine quadrant is similar to an analog computer which can be used to solve trigonometric equations. Physically, this tool consists of horizontal and vertical grids bound by two axes, the horizontal axis (*jaibul tamam*) starting from the central quadrant up to the beginning of the curve (*awal qaus*) indicated as 0°. The vertical axis (*sittini*) starting from the central quadrant until the end of the curve (*akhir qaus*) indicated by 90°. The radius of the two axes is the radius of the quadrant, measuring 60 units (R = 60 units = 1 radius). The value of a circular 'chord', R = 60 is the same value used in basic concepts of geometry and trigonometry since the Greek and Hindu age and in Islamic civilization, as explain in [5]. The reading of the horizontal and vertical grids down from the two axes is divided into sixty units, making its scale in units of sexagesimal.

The quadrant curve or '*qaus*', is a bow-shaped curve formed from the central quadrant. Grids which come down from the vertical axis to the curve become the adjacent side of a right triangle which form Pythagoras' theorem. [See Figure 1, this side is marked *Y*]. In the use of the sine quadrant, a grid reading on this scale from the angle of the curve to the vertical axis represents the value of the ratio of the sine functions. While the grids that are projected from the horizontal axis to the curve become the opposite side, marked by *X* in Fig. 1. Grid reading on this scale from the angle of curve of the horizontal axis represents the value of the cosine ratio functions. Each grid represents the ratio of horizontal and vertical angles which are read on the curve (0° to 90 °) and centered on the origin of the quadrant. In Figure 1, the angle θ of the triangle *POQ* is the angle formed by the ratio of the side adjacent to the opposite side, while the hypotenuse measures R = 60 units. Hence, the triangle *POQ* gives the value;

$$\sin\theta = \frac{X}{60} \tag{1}$$

$$\cos\theta = \frac{Y}{60} \tag{2}$$

In the use of the sine quadrant, the function of the cosine angle is shown as a function of the complementary angle of sine functions. If the angle is measured on the curve as θ, then the complementary angle is (90°- θ). Sine angle is read from the initial curve, starting at 0° and the ratio following the horizontal grid to the vertical axis. Whereas the cosine angle starting at the end of the curve at a value of 90° (read as 0° for cosine functions). However, if the cosine angle needs to be determined at the initial point of curve (starting at 0°), the ratio should be read from the horizontal axis using the vertical grid. This operations gives the correlations as follows;

$$\cos\theta = \sin(90° - \theta) \tag{3}$$

The intersection of the vertical and horizontal grid will give the angle of tangent functions.

Angle measurement scale on the curve of the quadrant is marked starting from 0° to 90°, can also be used to measure the scale of unit hours from 0 hour to 12 hours and the commencement date of the constellation starting from 01 degrees to 30 degrees for every month. All units of measurement scales can be read up to the estimation of the minimum value depending on the scale of the curve. For example, degree units with a small interval of 1°, can be estimated up to a value of $\frac{1}{8}$ (equivalent to 7.5 arc minutes). As for unit hours with a small interval equivalent to 4 minutes, the reading can be estimated up to 30 seconds.

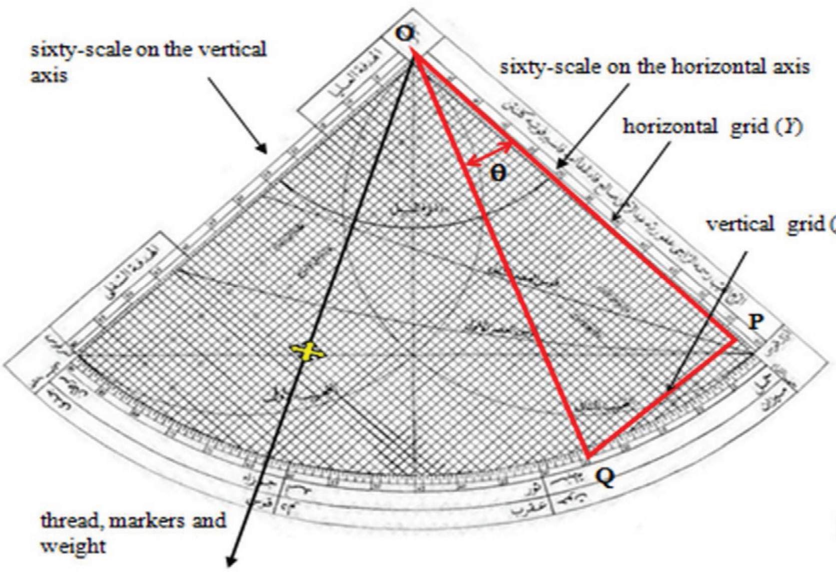


Figure 1: Formation of a triangle and measuring the angle of the sine quadrant

Elements and other components found in the design of the sine quadrant;

(a) Dates of the zodiac calendar on the same curve as the value of angle and time.

(b) Lines of the Sun's ecliptic which gives the coordinates of the Sun from the celestial equator. Changes in these coordinates correspond to the dates of the zodiac calendar.

(c) Two lines representing the ratio of tangent angle functions with gnomon representing 7 and 12 units. The 'gnomon' scale is used to alter the length of the shadow of an object relative to the Sun's angle of altitude on a tangent formula or vice-versa.

(d) Two semi-circle lines with a radius of 30 units of the scale grid drawn on the horizontal axis and vertical axis, from their respective midpoints.

(e) Finder or target to measure the altitude of objects including the Sun or for easier measurement of angles between two reference points.

(f) Thread and knot as a marker for values of the angles and the ratio during measurement and calculation operations. The weight is attached to the thread to ensure stretchability while marking, moving the thread and when taking the readings.

3. Operations Using Sine Quadrant

Generally, descriptive mathematics to solve trigonometric equations by using the sine quadrant are noted in the Malay World manuscripts as follows;

- Place the thread at an angle or ratio
- Move the thread to the second angle or ratio
- Obtain solutions for an operation

3.1 General Trigonometric Equations

As explained earlier in this paper, the sine quadrant is a mathematical instrument which is used to read the ratios of trigonometric functions and perform calculations. The angle read on the curve is described as '*qaus* ', while the ratio of sine of angle is described as '*jayyib*'. The angle is read on the curve with a scale from 0° to 90°, while the ratio is read according to a horizontal grid to vertical axis with a scale from 0 to 60 units. The relation between trigonometric function in sine quadrant can be explained as follows:

(a) The angle '*qaus*' 90° on the curve, which produces a ratio equal to 60 in the sexagesimal system is equivalent to 1 in the decimal system;

qaus 90° = *jayyib* 60 (in sexagesimal system) or sine 90° = 1 (in decimal system)

qaus 30° = *jayyib* 30 (in sexagesimal system) or sine 30° = 0.5 (in decimal system)

For every sine angle value read from the curve, the ratio is read respectively following the horizontal grid up to the reading on the vertical axis.

(b) The ratio of the sine angle read on the vertical axis can also transform into the angle which is read on the curve. It becomes the form of reciprocal of sine ($\sin^{-1}$);

jayyib 13 ½ = *qaus* 13° (in sexagesimal system), or

$\sin^{-1}(0.225) = \sin 13°$ (in decimal system)

(c) All other trigonometric functions such as tangent and cosine of an angle (example α), can be produced from the sine angle, namely;

$$\cos\alpha = \sin(90° - \alpha) \tag{4}$$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} \text{ or } \tan\alpha = \frac{\sin\alpha}{\sin(90°-\alpha)} \tag{5}$$

(d) To obtain an unknown angle of tangent (α), the right value must be in the form of ratio, simplified as:

$$\tan\alpha = \frac{X}{Y} \tag{6}$$

Where, X is the vertical scale and Y is the horizontal scale. If $\tan\alpha = X$, therefore there is no solution.

3.2 Forms of Equation Solution

(a) To obtain a solution of an unknown angle A from an addition of two trigonometric equations;

$$\sin A = \sin X + \cos Y$$
$$\sin A = \sin X + \sin(90° - Y)$$
$$\sin A = \textit{jayyib}\ X + \textit{jayyib}\ (90° - Y)$$
$$\sin A = \textit{jayyib}\ Z$$

Read the *jayyib X* and *jayyib* (90°-*Y*), add both to become *jayyib Z*. Read this *jayyib* total on the vertical axis and descend following the horizontal grid lines to the curve to obtain the angle of *A*. [See Figure2].

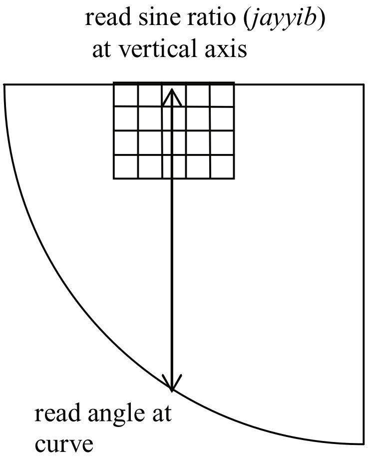


Figure 2 : Read sine ratio from value of angle at curve or vice-versa

(b) To obtain a solution of an angle from a multiplication of two trigonometric equations;
Form 1:

$$\sin A = \sin X \cos Y$$
$$\sin A = \sin X \sin(90° - Y)$$

(i) Put the thread on the vertical axis, place the markers on the value of the *jayyib X*. Then move the thread to the curve at the value of angle (90°-*Y*).

(ii) According the position of the marker, read the horizontal grid lines to curve to obtain the angle of *A*. [See Figure 3].

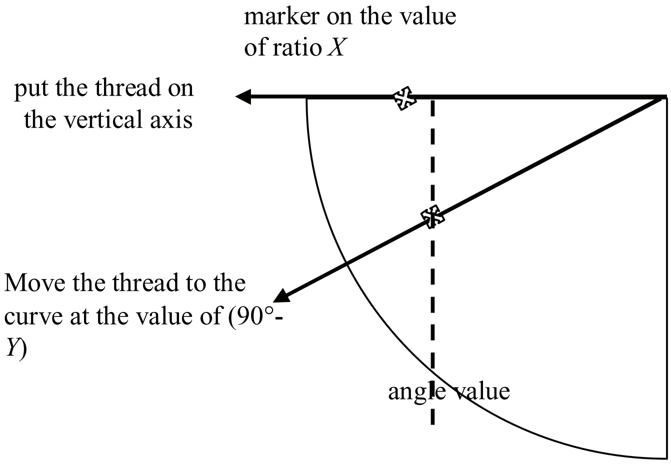


Figure 3 : Quadrant operations to solve the multiplication equations

Alternatively, the above equation can be solved by multiplying *jayyib X* and *jayyib* (90°-*Y*). If it is more than 60, divide it by 60 and take the quotient as *jayyib*. Read this *jayyib* on the vertical axis and obtain the angles of the curve following the horizontal grid line.

Form 2:

$$\sin A = \sin X \sin Y \cos Z$$

$$\sin A = \sin X \sin Y \sin (90° - Z)$$

(i) Put the thread on the vertical axis, place the marker on the value of *jayyib X*. Move thread to the curve at angle *Y*. Read the position marker following the horizontal grid lines to the vertical axis, to obtain the value of the *jayyib* (say *jayyib P*).

(ii) Put the thread on the vertical axis, place the marker on the value of *jayyib P*.

(iii) Move the thread to the curve at angle (90°-*Z*). According the position of the marker, read the horizontal grid lines to curve to obtain the angle of *A*.

(c) To obtain a solution of the equation of the quotient;

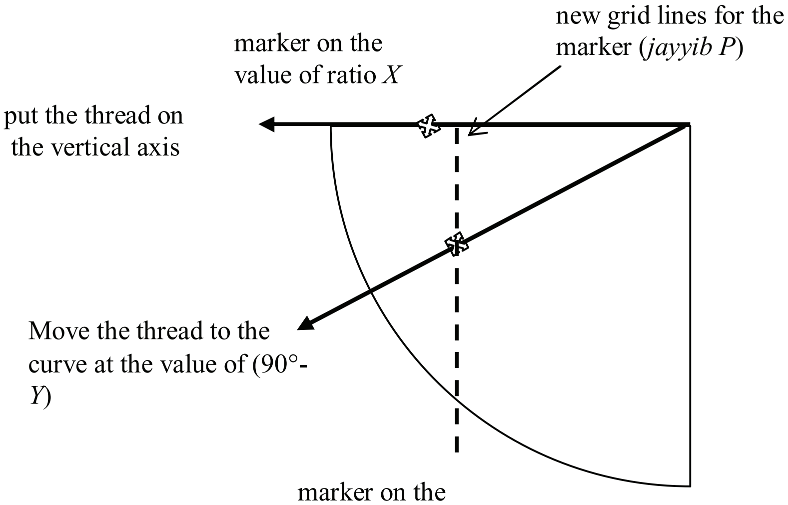
Form 1:

$$\cos A = \frac{\sin X \sin Y}{\sin Z}$$

(i) Put the thread on the vertical axis, place the marker on the *jayyib X*. Move thread to a curve at the point Y. Read the position marker in the horizontal grid lines to the vertical axis, to get the *jayyib* (say *jayyib P*).

(ii) Put the thread on the vertical axis with the marker at the *jayyib Z*.

(iii) Move the thread to the curve in the condition of the marker meets the grid line of *jayyib P*. Read the thread positon on the curve as angle *A* starting from 90º (because angle A is the cosine functions). [See Figure 4].



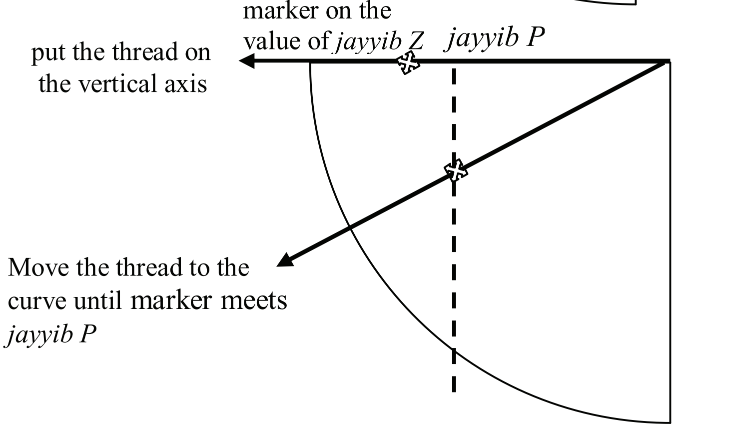


Figure 4 : Quadrant operations to solve the quotient equations

Form 2:

$$\tan A = \frac{\sin M \ \sin N}{\sin P \ \sin Q}$$

(i) Put the thread on the vertical axis with the marker on the *jayyib M*. Move thread to a curve at the value of *N*. Read the marker position in the horizontal grid lines to the vertical axis, to get the *jayyib* (say *jayyib X*).

(ii) Put the thread on the vertical axis with the marker at the *jayyib P*. Move thread to a curve at the angle value of *Q*. Read the position marker in the horizontal grid lines to the vertical axis, to get the *jayyib* (say *jayyib Y*).

(iii) Put the thread in the intersections point of a vertical grid lines (with value of *jayyib X*) with the horizontal grid lines (for the value of *jayyib Y*). Read the angle *A* at the position of the thread at the curve starting from 0º. [See Figure 5].

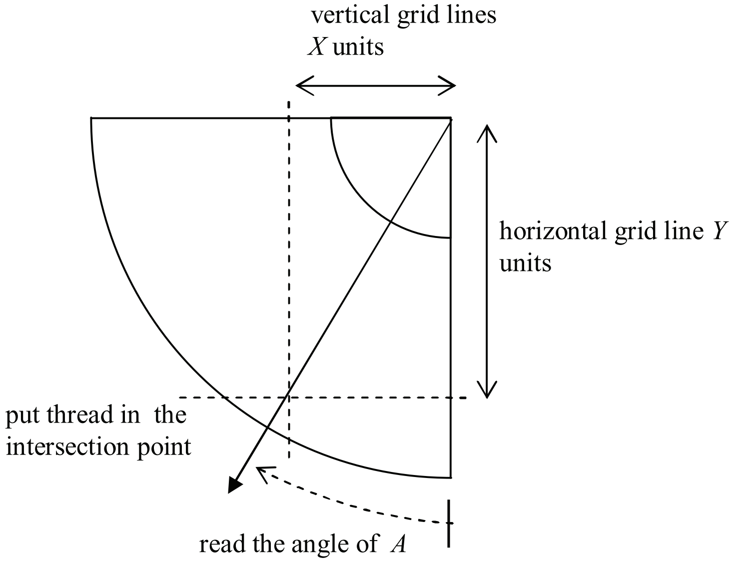


Figure 5: Quadrant operations to read angle for tangent functions

4. Conclusion

The sine quadrant is an effective mathematical tool used for the publication of trigonometric tables and solving trigonometric equations. Its design represents the basic idea of trigonometrical mathematics in the days of Islamic civilization which was brought to the Malay World. The basis of this mathematics is required for astronomical calculations, calculating prayer times, the direction of the qiblah and calendar. It is also used in mathematical solutions for determining the parameters of geoinformation, such as the latitude and longitude of the earth, measuring the height of the mountain, measuring the area of a particular region and well depth. Based on this study, it is found that scholars of astronomy in the Malay World have used the equipment with the concept of geometry, trigonometry and astronomy long before the arrival of European colonists. The accuracy and efficiency of the sine quadrant depends on its design and efficiency of use. Efforts should be made to introduce the sine quadrant as a mathematical instrument in trigonometric topics taught at lower secondary school level.

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